Placing Landmarks Suitably for Shape Analysis by Optimization

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Landmarks

Landmarks:

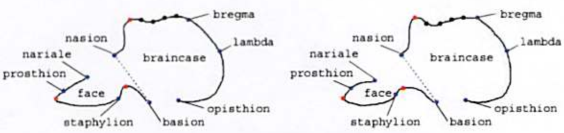
- are a finite number of points on the contour of an object to compare the shapes of objects.
- are points of correspondence that matches between and within objects of the same class, with the correspondence shown by a label.
- can be classified into three types,
  - anatomical landmarks are assigned by experts in some biologically meaningful manner.
  - mathematical landmarks are points that indicate some mathematical or geometrical feature.
  - pseudo-landmarks are not classified into the above two landmark types.

Landmarks: An Example

anatomical landmarks are assigned by experts in some biologically meaningful manner.

mathematical landmarks are points that indicate some mathematical or geometrical feature.

pseudo-landmarks are not classified into the above two landmark types.



Landmarks on monkey skull midlines:
• is anatomical, • is mathematical, and • is pseudo.

Landmark-based Analysis

Statistical Shape Analysis:

- is to compare shapes by exploiting invariant statistics under similarity transformations (translation, rotation, and isotropic scaling).
- is based on the placement of landmarks.
- falls into a classification problem over a hyper-sphere spanned by the landmarks.

☞ How should we place the landmarks on the contour of shape ?

Problem in Placing Landmarks:

- anatomical landmarks require expert knowledge about an object.
- mathematical landmarks require several assumptions about the contour curves of an object (e.g., a high curvature point depends on the assumption that the points are ordered and the curve can be represented as a twice differentiable function.).

☞ Not all applications satisfy these assumptions in practice !

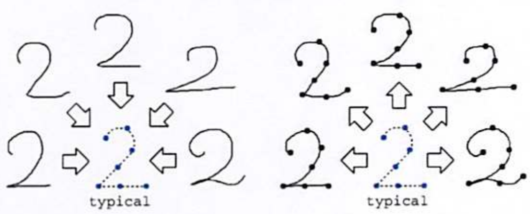
Aim and Outline

Aim:

- is to give an automatic placement of landmarks without these assumptions.

Our Method:

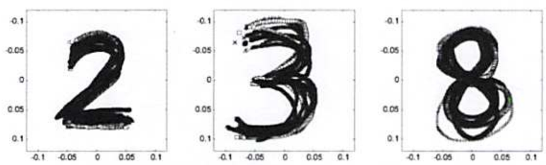
1. assumes that there exists a typical shape and obtain the landmarks of the typical shape (but not the typical shape itself).
2. places landmarks according to the landmarks.



Preprocessing by Similarity Transformations

Preprocessing:

- is to center a contour by translation, superimpose it by rotation, and normalize it by isotropic scaling.
- A set of points of the preprocessed contour is denoted by $\mathcal{X}_k$, $k = 1, \ldots, K$.



Points of the preprocessed contours of "2", "3", and "8" classes: the number of preprocessed contours in each class is $K = 10$.

Objective Function

The landmarks of the typical shape of the same class objects fall into the solutions of an optimization problem.

Objective Function

For any configuration of n landmarks $(x_1, y_1), \ldots, (x_n, y_n) \in \mathbb{R}^2$,

$$f(x_1, y_1, \ldots, x_n, y_n) \triangleq \sum_{i=1}^{n} \iint_{A_i} d\left((x_i, y_i), (x, y)\right) g_a(x, y)\, dxdy$$

where for any $i = 1, \ldots, n$,

- A_i is the Voronoi region of site (x_i, y_i)
- d is the Euclidean distance between two coordinates
- $g_a : \mathbb{R}^2 \to \mathbb{R}$ is the kernel function for any $a > 0$

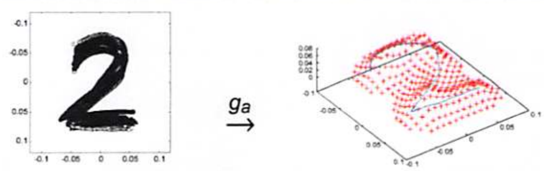
Kernel Function

Kernel Function

For any $a > 0$,

$$g_a(x, y) \triangleq \frac{1}{K} \sum_{k=1}^{K} \sum_{(x', y') \in \mathcal{X}_k} \exp\left(-a d\left((x, y), (x', y')\right)\right)$$

where $\mathcal{X}_k$ is the set of points in the k-th preprocessed contour for any $k = 1, \ldots, K$.



Preprocessed contours $\mathcal{X}_1, \ldots, \mathcal{X}_{10}$ $\xrightarrow{g_a}$ Kernel function with $a = 90$

Optimization Problem

- To avoid that some landmarks go to the same point, impose
  Constraint For any $L > 0$,

$$\forall i, j \ [d((x_i, y_i), (x_j, y_j)) \geq L, \ i \neq j]$$

- In summary, the n landmarks of the typical shape of the same class objects fall into the solutions of the following constrained optimization problem.

Optimization Problem

minimize: $f(x_1, y_1, \dots, x_n, y_n)$

$$= \sum_{i=1}^{n} \iint_{A_i} d((x_i, y_i), (x, y)) \, g_a(x, y) \, dxdy$$

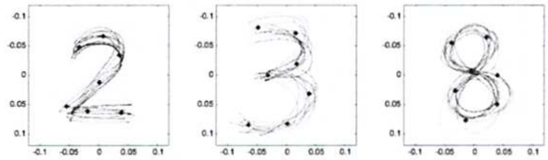
subject to: $\forall i, j \ [d((x_i, y_i), (x_j, y_j)) \geq L, \ i \neq j]$

Results: Landmarks of Typical shape

❑ **Input and Setting:**

- We employed $K = 10$ sets of preprocessed contour points for each of "2", "3" and "8"-classes
- Setting
  - Number of landmarks was $n = 7$
  - Parameter of the kernel function was $a = 90$
  - Parameter of the constraint was $L = 0.03$

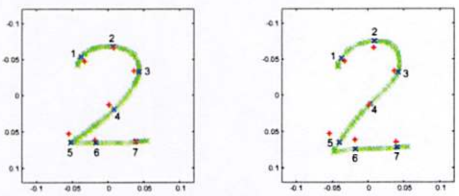
❑ **Configuration of landmarks:** The following landmarks were obtained by solving the optimization problem by a gradient method.
☞ The landmarks look like each class typical shape !



Landmarks of "2", "3" and "8"-class typical shapes

Placing Landmarks

- Landmarks of a preprocessed contour are the closest points from the landmarks of the typical shape given by the optimization problem.
- Landmarks of different preprocessed contours, based on the same landmark, have the same label.



Closest points from the landmarks of the typical set:
× is the closest point from a landmark of the typical set +,
and × is a point in the preprocessed contour.

Criterion for Evaluating Landmark Configuration

- We used the mean of the full Procrustes distances within the same class or between different classes.
- We expect that the mean is small within the same class, while the mean is large between different classes

Mean of the Full Procrustes Distances

For any class l, l',

$$\frac{1}{K^2} \sum_{i=1}^{K} \sum_{j=1}^{K} D_F\left(Y_i^{(l)}, Y_j^{(l')}\right),$$

where

- D_F is the full Procrustes distances (the minimum distance under similarity transformations)
- $Y_i^{(l)}$ is the configuration matrix made by the preprocessed contour $\mathcal{X}_i$ of l class

Results: Landmark Configuration

❑ **Mean of the full Procrustes distances:**

		class l'		
		"2"	"3"	"8"
class l	"2"	0.11	0.75	0.79
	"3"	0.75	0.14	0.94
	"8"	0.79	0.94	0.12

- The full Procrustes distances are small within the same class (see the cases of $l = l'$), while those are large between the different classes (see the cases of $l \neq l'$)
- Our method is useful in shape analysis to give landmarks well on the contours of the same class objects

Summary

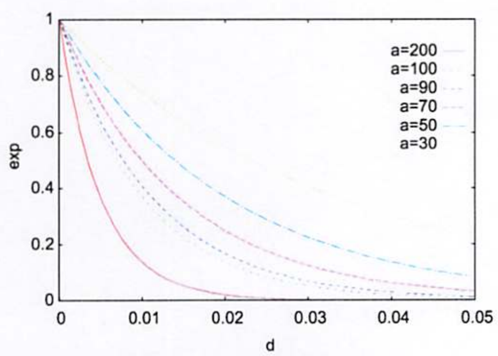
❑ **Summary:**

- We presented a method of how to place landmarks on the contours of the same class objects when the landmarks are not available.
- Our method provides an automatic placement of landmarks without an expert about the object or mathematical assumptions on the contour curves.

❑ **Future Works:** How should we find ...

- a good parameter of the kernel function a ?
- a good number of landmarks n ?

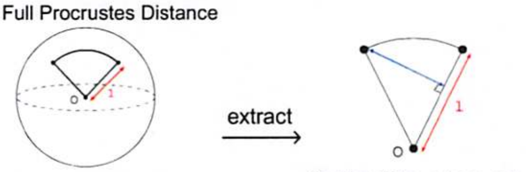
Appendix: Kernel Function



☞ The x- and y-axes are d and $\exp(-ad)$, respectively.

Appendix: Full Procrustes Distance

- Full Procrustes Distance



Sphere spanned by landmark configurations

Section of the sphere and the full Procrustes distance

- Volume of Unit Sphere V_n in n Dimensions (H. Maehara, *Geometry of Circles and Spheres*, Asakura Pub., 1998.)